INGENUE v.2. Project : The theoretical framework

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1 Introduction

The first version of the INGENUE model describe a multi-region, world model, in the spirit of those developed by Obstfeld and Rogoff [1996, chap. 3], in which the structure of each regional economy is similar to that of other applied, OGGE models, such as Auerbach and Kotlikoff [1987], except that labor supply is exogenous.
The world was divided into six regions, each of which is made of three categories of economic agents: the households, the firms, and a PAYG retirement pension system. There is only one good, and only one financial asset, which is an ownership stake in the firms' productive capital ; both of them are freely traded on perfectly competitive world markets. There is no money and hence only two relative prices in each region: the (real) wage rate accruing to local, internationally immobile, workers; and the single (real) price of financial assets, both expressed in terms of goods, which may be chosen for *numéraire*. Hence, the various regions of the world are economically and financially perfectly integrated and there is only one world market for goods and one for financial assets.
We have performed various work with this first model (among them Ingenue [2001], Ingenue [2002a], Ingenue [2002b]) but the model soon appear to be irrealistic in some of its outcome. So we decide to built a new version of this model.

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2 Ingenue 2. vs Ingenue 1. : What's New?

1. Demographics : the World is now divided in 10 regions. In order to make our own demographic projections we have built a population projection model based upon UN coefficient methods.

2. We now assume uncertainty in lifetime expectancy at individual level. At the macroeconomic level there is still no uncertainties.

3. International trade of commodities : In order to deal with relative price movements of foreign and domestic goods we assume that the different countries produce, different imperfectly susbstituables intermediate goods as in Backus et Al. (1995). This will imply the existence of real effective exchange rates between the different regions. Where the main determinants of exchange rates are the relative productivity in the two productive sector as in the standard view developed since Obstfeld and Rogoff works (i.e. the famous *Balassa-Samuelson effect* that is predominant in long run explanations of difference in real exchange rates).

4. We modelize region-specific interest rates to debtor that differ from the unique world interest rate to creditor by imposing an *ad hoc* convex function of the region own ownership ratio.

5. Calibration improvements : both unintended and voluntary bequests (based upon a bequest motive), age-specific labor participation rates (exogenous), age-specific human capital (exogenous), ...

3 Demographics

In this new version of the Ingenue model the World is now divided in 10 regions (instead of 6 regions in the first version of the model), according to geographical criteria :

1. **"Western Europe" :** 'Channel Islands', 'Denmark', 'Finland', 'Iceland', 'Ireland', 'Norway', 'Sweden', 'United Kingdom', 'Greece', 'Italy', 'Malta', 'Portugal', 'Spain', 'Austria', 'Belgium', 'France', 'Germany' (East + West), 'Luxembourg', 'Netherlands', 'Switzerland'.

2. **"Eastern Europe" :** 'Estonia', 'Latvia', Lithuania', 'Bulgaria', 'Czech Republic', 'Hungary', 'Poland' 'Romania', 'Slovakia', 'Slovenia', 'Albania', 'Bosnia and Herzegovina', 'Croatia', 'TFYR Macedonia', 'Yugoslavia'.

3. **“North America” :** ’Canada’, ’United States of America’, ’Australia’, ’New Zealand’, ’Melanesia’, ’Fiji’, ’New Caledonia’, ’Papua New Guinea’, Solomon Islands’, ’Vanuatu’, ’Micronesia’, ’Guam’, ’Polynesia’, ’French Polynesia’, ’Samoa’.

4. **“South America” :** ’Argentina’, ’Bolivia’, ’Brazil’, ’Chile’, ’Colombia’, ’Ecuador’, ’French Guiana’, ’Guyana’, ’Paraguay’, ’Peru’, ’Suriname’, ’Uruguay’, ’Venezuela’, ’Belize’, ’Costa Rica’, ’El Salvador’, ’Guatemala’, ’Honduras’, ’Mexico’, ’Nicaragua’, ’Panama’, ’Bahamas’, ’Barbados’, ’Cuba’, ’Dominican Republic’, ’Guadeloupe’, ’Haiti’, ’Jamaica’, ’Martinique’, ’Netherlands Antilles’, ’Puerto Rico’, ’Saint Lucia’, ’Trinidad and Tobago’.

5. **Japan**

6. **“Mediterranean World” :** ’Algeria’, ’Egypt’, ’Libyan Arab Jamahiriya’, ’Morocco’, , ’Tunisia’, ’Western Sahara’, ’Armenia’, ’Azerbaijan’, ’Bahrain’, ’Cyprus’, ’Georgia’, ’Iraq’, ’Iran’, ’Israel’, ’Jordan’, ’Kuwait’, ’Lebanon’, ’Occupied Palestinian Territory’, ’Oman’, ’Qatar’, ’Saudi Arabia’, ’Syrian Arab Republic’, ’Turkey’, ’United Arab Emirates’, ’Yemen’. ’Turkmenistan’, ’Uzbekistan’ ’Kyrgyzstan’

7. **“Chinese World” :** ’China’, ’Democratic People’s Republic of Korea’, ’Mongolia’, ’Republic of Korea’, ’Brunei Darussalam’, ’Cambodia’, ’East Timor’, ’Lao People’s Democratic Republic’, ’Myanmar’, ’Philippines’, ’Singapore’, ’Thailand’, ’Viet Nam’.

8. **“Africa” :** ’Burundi’, ’Comoros’, ’Djibouti’, ’Eritrea’, ’Ethiopia’, ’Kenya’, ’Madagascar’, ’Malawi’, ’Mauritius’, ’Mozambique’, ’Réunion’, ’Rwanda’, ’Somalia’, ’Uganda’, ’Tanzania’, ’Zambia’, ’Zimbabwe’, ’Angola’, ’Cameroon’, ’Central African Republic’, ’Chad’, ’Congo’, ’Democratic Republic of the Congo’, ’Equatorial Guinea’, ’Gabon’, ’Botswana’, ’Lesotho’, ’Namibia’, ’South Africa’, ’Swaziland’, ’Benin’, ’Burkina Faso’, ’Cape Verde’, ’Côte d’Ivoire’, ’Gambia’, ’Ghana’, ’Guinea’, ’Guinea-Bissau’, ’Liberia’, ’Mali’, ’Mauritania’, ’Niger’, ’Nigeria’, ’Senegal’, ’Sierra Leone’, ’Togo’. ’Sudan’

9. **“Russian World” :** ’Belarus’, ’Russian Federation’, ’Ukraine’. ’Kazakhstan’, ’Republic of Moldova’,

10. **“Indian World” :** ’India’, ’Afghanistan’, ’Bangladesh’, ’Bhutan’, ’Maldives’, ’Nepal’, ’Pakistan’, ’Sri Lanka’, ’Tajikistan’, ’Indonesia’, ’Malaysia’.

The period of the model is set to five years. In each region z, the economy is populated by overlapping generations of one-sex agent who may no live longer than 100 years. The number of people of age a at time t is denoted by $L_a^z(t)$ (for any variable, a subscript a denotes age and an argument t in parentheses denotes calendar time). At date t the number of "births" (individuals between 0 and 4 years old) is then denoted by $L_0^z(t)$ and total population alive at time t is $L^z(t) = \sum_{a=0}^{100} L_a^z(t)$.

Population evolution are exogenously calculated according to a standard population projection method on the basis of historical and prospective UN data. We have aggregated population structure, with the UN data from 1950 to 1995, over countries to build Ingenue's regions (see upper). Then we have projected fertility and mortality trends (for both sexes) at the region-aggregate level, this together with initial population structures in 1995, allow us to obtain population evolution in the future from 2000 until the ending date of the model. We implicitly assume that there is no migration flows in the future. With some usual population projection methods, we construct evolution of mortality and fertility tables on the only basis of life expectancy and global fertility rates evolutions in the future.

3.1 Mortality

People can die before 100 year ; let s_a the conditional probability of surviving between age a and age $a+1$, the number of age $a-1$ people then changes as (recall that a = 5 means sum of the cohorts from 5 to 9 years):

$$L_a^z(t) = s_{a-1}^z(t-1) \cdot L_{a-1}^z(t-1) \qquad a = 5 \ldots 100 \tag{1}$$

$\prod_{i=0}^{a-1} s_i(t+i)$ is then the unconditional probability of being alive at age "a" when born at date t. For population projection we then need some process to describe evolution of $\{s_{a-1}^z(t-1)\}_{a>0}$ for $t = 2000, \ldots, T$ (for both sexes). For this we first have to precise beginning and ending mortality tables. Beginning table is given from UN data between years 1995 and 2000. Ending table are chosen among UN "typical" long run mortality tables (from Coale and Demeny, 1966). We then have to choose a date when the convergence to the long run table will be achieve and a process for convergence between initial date and this date. According to UN methods we extrapolate future mortality tables on the basis of a expected trend for life expectancy. We adopt a linear process of convergence.

3.2 Fertility Process

With deterministic population we have the number of births equal to $L_0^z(t) = \sum_{a=15}^{50} f_a^z(t) L_a^z(t)$ (here L is only female population), where f_a^z are the average age-specific fertility rates. As a matter of fact, we assume following UN projections that women fertility occurs only between 15 and 50 years old.

4 The household sector

Individuals are assumed to become adults when they turn 20. During any period, the household sector is then made of 21 overlapping cohorts of "adults", of age between 20 and 105, and 4 cohorts of "young". Adults may no stay in the labor force after a legal maximal mandatory retirement age $\bar{r}^z$. Economic decisions are their consumption and saving decisions, made with perfect foresight at the beginning of their adult life. Between 15 and 50 yrs. adults are supposed to give birth to children, according to the fertility calendar. Children are dependent until they turn 20, they consume with a cost per child that is supposed to be proportional to the parents consumption.

Labor supply is assumed to be exogenously given as the age-specific rate of participation to labor market : ta_a^z. We take ILO data and projections to characterize activity from 1950 until 2015 and assume that after this date participation rates remain fixed at their 2015 level. According to this database people may work since the age of 10 so we will take into account children labor income to the budget constraint of their parents.

The intertemporal preferences of a new entrant on working-life are given by the following life-time utility function over uncertain streams of consumption c_a^z and leaving a voluntary bequest H^z to their children at an age T (if they survive until this age)[1] :

$$U^z(t) = \sum_{a=20}^{100} \rho^{a-20} \prod_{i=20}^{a-1} s_i^z(t+i) \frac{\eta}{\eta - 1} c_a^z(t+a-20)^{\frac{\eta-1}{\eta}} + \rho^T \prod_{i=20}^{T} s_T^z(t+T-1) V\left(H^z(t+T)\right) \quad (2)$$

where ρ is the psychological discount factor[2], C_a is consumption at the age a ; η is the intertemporal substitution rate and $V(\cdot)$ is the instantaneous

[1]Usually in these kind of model the age T is the biological limit to life (here 105 yrs.) but in order to imply a realistic pattern of inheritance among the children of deceased households, we assume that T is at the exogenous expected lifetime given by the demographic projections.

[2] Notice that the effective discount rate is equal to $\prod_{i=20}^{a-1} s_i^z(t+i) \frac{\eta}{\eta-1} \rho^{a-20}$, mean-

utility of bequest, so agent has some felicity from leaving a bequest but it is independent of the future stream of the consumption that the children draw from this bequest (*warm glow* altruism). This bequest behaviour is mainly adopted to calibration issues (empirically savings for life-cycle can only explains a part of saving motives).
At any given period, the budget constraint is(with additional constraints $S_{15} = 0$ and $S_{105} \geq 0$) for a = $a_0, \ldots, 105$:

$$\begin{aligned} & \tau_a^z(t) P_f^z(t) C_a^z(t) + P_f^z(t) S_a^z(t) = Y_a^z(t) + R^z(t) P_f^z(t) S_{a-1}^z(t-1) \\ + \quad & P_f^z(t)(B_a^z(t) + b_a^z(t)) \end{aligned} \tag{3}$$

$$Y_a^z(t) = \begin{cases} \zeta_a^z(t) + (1-\theta^z(t)) W(t)^z h_a(t) \epsilon_a & \text{for} \quad a < \underline{r}^a \\ (1-\theta^z(t)) W(t)^z t a_a(t) \epsilon_a + (1 - t a_a(t)) \pi_a^z(t) & \text{for} \quad \underline{r}^a \leq a < \bar{r}^a \\ \pi_a^z(t) & \text{for} \quad a \geq \bar{r}^a \end{cases}$$

where due to life uncertainty $B_a^z(t)$ are unintended bequests that are taken as lump-sum by individuals while $b_a^z(t-1)$ is the inherited assets received from voluntary bequests of their deceased parents ; S_a^z denotes the stock of assets held by the individual at the end of age a and time t, $R^z(t) \cdot S_a(-1)$ is financial income (domestic real return on assets holdings times wealth), τ_a is the age-specific equivalence scale that takes into account the direct and indirect private costs of child-rearing, and Y_a is the non-assets net disposal income. For full-time active years ($a \in [a_0, \underline{r}^a[$) it is simply equal to the net labor income after social security taxes (at rate θ), where W is the real wage rate per efficient unit of labour at time t. When agent is partly retired ($a \in [\underline{r}^a, \bar{r}^a[$) he also receives a pension benefit P_a for the unworked hours. And when he is full-time retired ($a \in [\bar{r}^a, a_T)$) he only receives the pension benefit . In this paper, pension benefit is assumed to be age dependant first in order to take into account the indexing pension rule and second to specify some kind of specific rule for pension before $\bar{r}^a$ the maximum mandatory retirement age.
$P_f^z(t)$ is the price of the domestic final good and then $R^z(t)$ is expressed in units of this final good.
The τ_a^z term is the age-specific equivalence scale, it takes account the direct and indirect private costs of child-rearing. In order to calculate this relative cost of child-rearing for each cohort we need first to know the age distribution of children for each parent (from their past fertility behaviour) and second we need the age "c" (for child) equivalence scale of children β^c, which will be assume here to be constant :

ing that agents only care of their future as long as they stay alive. In other words the expectation takes into account that the agent can die before 105 yrs. old

$$\tau_a^z(t) = 1 + \sum_{c=\max(0,a-50)}^{\min(15,a-15)} \beta^c \cdot L_a^{c,z}(t) \qquad a = 20, \ldots, 50 + 20 - 5 \tag{4}$$

where the average number L_a^c of children of age c raised by cohort of age a can be recover from fertility evolution (given the fertility calendar and the early death of both children and parents). For simplicity, the children depending of parents younger that 20 years old are assumed to be "allocated" between the adults that have same age children (allocation with age-specific weights)[3]. $\zeta_a^z(t)$ is the labour income that children bring to their parents resources during their childhood (calculated in the same spirit that costs of children-rearing).

An agent's earning ability is assumed to be an exogenous function of its age. These skill differences by age are captured by the efficiency parameter ϵ_a which changes with age in a hump-shape way to reflect the evolution of human capital. For simplicity, we assume that this age-efficiency profile is time-invariant and is the same in all region. In the baseline case we adopt Miles (1999) human capital profile's estimation (for UK) and normalize ϵ_a such that $\epsilon_{20} = 1$.

As in Yaari [1965] we assume that, though individuals are uncertain about the length of their life, the population is large enough to ensure aggregate certainty over the population of each cohort. Contrary to Yaari [1965] there is no insurance companies or perfect and fair annuities market such that mortality risks are pooling within the same cohorts to cover the eventuality of early death[4]. We do not retain this assumption because it seem rather unrealistic for most of industrialized countries with regards to the actual volume of such contracts as documented in Gaudemet [2001]. Mahieu and Sédillot [2000] explain this low trading in contracts by their "imperfect nature" that can't solve the *self-selection problems.* We then have to precise how is distributed the assets of dying people. For simplicity we follow Imrohoroglu [1998]'s assumption that accidental bequests are taxed at 100 % and lump-sum rebated among all the survivors by government. As discussed in Bohn [1999] the assumptions made on unintended bequest, their distribution, and perfect annuities markets have to be much more cautious when one considers random life survival rates which is not the case here. Even if

[3]Being more precise will need to conserve the distribution of child with respects to their grand-parents and will complicated in an useless way the number of state variables in the system.

[4]The Yaari's assumption is retained in most of EGCM analysis for France (Chauveau and Loufir [1997] or Docquier, Liégeois, Loupias and Crettez [2002]) as well as for other countries (Rios-Rull [1996] for US, Broer and Westerhout [1997] for Dutch, . . .

the consequences of such choices are smaller here we have to precise that the distribution of accidental bequests will nevertheless matter, for instance if part of bequest are distributed to younger active cohorts they appear to be more sensitive to capital return risk than they would be in their absence. So the lump-sum accidental bequest received by any adult cohorts at the end of period t are:

$$B_a = \frac{\sum_{a=a_0}^{100}(1-s_a)L_a S_a}{\sum_{a=a_0}^{104} s_a L_a} \qquad \forall a \geq a_0 \tag{5}$$

i.e. each surviving adult receive an equal lump-sump share of unintended bequest of the dying people.
At the equilibrium the first order conditions to the households will yield, $\forall z$:

$$\frac{c_{a+1}^z(t+1)}{c_a^z(t)} = \left[\rho s_a^z R(t) \frac{\tau_a^z(t)}{\tau_{a+1}^z(t+1)}\right]^\eta \qquad a \in [20, 100[\tag{6}$$

$$c_{100}^z(t)^{-\frac{1}{\eta}} = V'\left(b^z(t)\right) \tag{7}$$

Voluntary bequests are distributed to children according to the fertility calendar of their deceased parents. Taking Blinder (1975) functional form for V, we obtain a simple linear relation for (7) : $c_T^z = \Psi H^z$, where Ψ indicates the degree of altruism. At the equilibrium the sum of voluntary bequest will be equal to the bequests received :

$$L_T^z(t)H^z(t) = \sum_{a=T-15}^{T-50} L_a(t)b_a(t) \tag{8}$$

In our international context households can choose the region we want for invest his wealth. So we need another equilibrium equation that characterizes the trade off between distinct assets :

$$R^z(t+1) = R^\star(t+1)\frac{P^z(t)}{P^z(t+1)} \tag{9}$$

where $R^\star(t+1)$ is the unique world interest factor expressed in the world *numéraire* (see later for details on this point), the condition 9 means that if a region z household save one unit in is domestic asset (capital) it will yield $R^z(t+1)$ in domestic currency at the next period, if he choose to invest on international market he will receive in terms of domestic capital goods $R^\star(t+1)\frac{P^z(t)}{P^z(t+1)}$. Equilibrium will imply that the two return will equalize.

4.1 The public sector

The public sector is reduced to a social security department; it is a pay-as-you-go (PAYG) public pension scheme, that is supposed to exist in all regions of the world. It is financed by a payroll tax on all labor incomes and pays pensions to retired households. The regional PAYG systems operate according to a defined-benefit rule: pensions π paid to individual retired are a fraction - or replacement rate (κ) – of the current average (net of tax) wage. We assume a time-to-time balanced-budget rule ($\forall i$):

$$\frac{\theta^z}{1-\theta^z} = \kappa \frac{\sum_{a \geq \underline{r}^a} (1 - ta_a) L_a}{\sum_{a \geq \bar{r}^a} ta_a L_a} \tag{10}$$

In the baseline case, the regional age $\underline{r}^a$ of minimum legal retirement age and the ratio κ are fixed, then payroll tax rates θ are endogenously determined by (10).

5 Production side

5.1 Intermediary good production sector

Each zone z specializes in the production $Y^{I,z}$ of a single intermediary good labelled I, where subscript ^{z} indicates that the specific nature of this good lies in its region of origin . Production in period t takes place with a CRS-CD production function using capital stock $K^z(t-1)$ installed at the beginning of the period in the country and the full domestic labor force N^z, $\forall z$:

$$Y^{I,z}(t) = F^{I,z}\left(K^z(t-1), N^z(t)\right) = A^{I,z}(t)\left(K^z(t-1)\right)^{\alpha}\left(N^z(t)\right)^{1-\alpha} \qquad 0 < \alpha < 1 \tag{11}$$

With this formulation $Y^{I,z}$ also denotes GDP in the country z in terms of the local intermediary good. The cash-flow of the representative domestic firm of the intermediary sector (in terms of the world *numéraire*) :

$$\begin{aligned} CF^z(t) &= P_I^z(t)Y^{I,z}(t) - w^z(t)L^z(t) - P_f^z\delta^z(t-1))I^z(t) \qquad \text{for t = 0} \quad (12) \\ CF^z(t) &= P_I^z(t)Y^{I,z}(t) - w^z(t)L^z(t) - P_f^z(1-\delta^z(t))K^z(t-1) \qquad \text{for t ¿ 0} \end{aligned}$$

where $P^{I,z}$ is the price of the domestic intermediate good ; $I^z(t) = K^z - K^z(t-1)(1-\delta^z(t))$ is the gross investment expenses in domestic final good and $\delta^z(t)$ is the rate of economic depreciation. In time 0 the present value of the firm is equal to :

$$\Pi^z(0) = CF^z(0) + \sum_{t=1}^{\infty} \frac{CF^z(t)}{\prod_{s=0}^{t} R^\star(t)} \tag{13}$$

Let denote $k^z(t-1) = K^z(t-1)/N^z(t)$ the capital-labor ratio, the maximization of the firm value will imply at the equilibrium $(\forall t)$:

$$\begin{aligned} R^\star(t+1)\frac{P^z(t)}{P^z(t+1)} + \delta^z(t+1) &= \frac{P_I^z(t+1)}{P_f^z(t+1)}\alpha A^{I,z}(t+1)\left(k^z(t)\right)^{\alpha-1} && (14) \\ w^z(t) &= P_I^z(t)(1-\alpha)A^{I,z}(t)\left(k^z(t-1)\right)^{\alpha} && (15) \end{aligned}$$

5.2 A "trick" to modelize imperfection on world financial market

For a world model to be realistic the world asset capital market have to be imperfect. Because sources of imperfection and asymmetries in financial markets are various and uneasy to modelize with rigourous microfoundations in such a large scale model as Ingenue we adopt the following *ad hoc* formulations for δ^z the region-specific rate of economic depreciation, with $\varepsilon > 0$, $\forall z$:

$$\delta^z(t) = 1 - (1-\bar{\delta})\left[Min\left\{\frac{A^z(t)}{K^z(t)};1\right\}\right]^{\varepsilon} \tag{16}$$

where $A^z(t) = L_T^z(t)H^z(t) + \sum_{a=a_0}^{104} s_a L_a S_a$ is the aggregate wealth over the overall cohorts in region z (actually this financial wealth is equal to the sum of the region capital stock and the net assets on the rest of the world). This equation then indicates that capital invested in a region z is depreciate more than average when the region is a net debtor of the rest of the world. In other world the net-of-depreciation return from capital invested in indebted regions are, other things being equal, lower than in creditor regions. With this formulation the domestic cost of capital for the intermediary sector may be greater than the world cost but neither lower.

5.3 Final good production sector

The region-z-specific Final good (consumption and investment) $Y^{F,z}$ is a composite of "World" (see below) and "Domestic" intermediary goods, respectively denoted $M^{\star,z}$ and $X^{D,z}$, according to the following CRS-CES technology, where $\sigma > 0$ denotes the elasticity of substitution, $\forall z$:

$$Y^{F,z}(t) = A^{F,z}(t)\left[\omega_z^{\frac{1}{\sigma}}\left(X^{D,z}(t)\right)^{\frac{\sigma-1}{\sigma}} + (1-\omega_z)^{\frac{1}{\sigma}}\left(M^{\star,z}(t)\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},\ \omega_z \in [0,1] \tag{17}$$

This CES combination of external and internal good to produce domestic final good is a *reminescence* of *Armington [1969] aggregator*. Taking prices as given and competitive behaviour the producer determines $X^{D,z}$ and $X^{W,z}$ that minimises current profit: $P_F^z(t)Y^{F,z}(t) - P_I^z(t) \cdot X^{D,z}(t) - P^\star(t) \cdot M^{\star,z}(t)$ subject to (17), where P_I^z is the price of the home-specific intermediate good and $P^\star$ is the price of the world intermediate good, both expressed in terms of the specific final good. The static maximization problem of the competitive firm give the following first order conditions :

$$X^{D,z}(t) = \omega_z \left(\frac{P^{I,z}(t)}{P^{F,z}(t)}\right)^{-\sigma} \frac{Y^{F,z}(t)}{(A_{F,z}(t))^{1-\sigma}} \tag{18}$$

$$M^{\star,z}(t) = (1-\omega_z) \left(\frac{P^\star(t)}{P^{F,z}(t)}\right)^{-\sigma} \frac{Y^{F,z}(t)}{(A_{F,z}(t))^{1-\sigma}} \tag{19}$$

where $P^{F,z}$ is the aggregate price index that can be shown to be equals to the following value at the equilibrium:

$$P^{F,z}(t) = \frac{\left[\omega_z \left(P^{I,z}(t)\right)^{1-\sigma} + (1-\omega_z)\left(P^\star(t)\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}}{A_{F,z}(t)} \tag{20}$$

6 The fiction of a world producer of an homogenous world intermediary good

In order to simplify the exchanges of intermediary goods between regions of the world we assume that there exist a fictive world producer that uses region-specific intermediary goods in quantities $X^{\star,z}$ in order to produce a specific world intermediate good $Y^\star$ according to the following *Armington aggregator* function :

$$Y^\star(t) = \left[\sum_z \gamma_z^{\frac{1}{\mu}} X^{\star,z}(t)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} \tag{21}$$

where γ_z is a weighted coefficient that represents the importance of the region z specific intermediary good in the world trade market (for instance think of oil producer regions, with respect to traditional theoretical trade determinants their share in world trade would be relatively small so one way to reproduce their actual huge trade balance is to allow an important size to this parameter). This agent search $X^{\star,z}$ that maximizes $P^\star(t)Y^\star - \sum_z P_I^z(t) \cdot M^{\star,z}(t)$, let define $e^z(t) = \frac{P_I^z(t)}{P^\star(t)}$ subject to (21) which gives the following solution (at the equilibrium) :

$$X^{\star,z} = \gamma_z \left(\frac{P^{I,z}}{P^\star}\right)^{-\mu} Y^\star \tag{22}$$

Again it can be shown that at the equilibrium $P^\star$ equals to :

$$P^\star = \left[\sum_z \gamma_z \left(P^{I,z}\right)^{1-\mu}\right]^{\frac{1}{1-\mu}} \tag{23}$$

7 A world General equilibrium

Let $C^z = \sum_{a\leq 20} \tau_a^z L_a^z c_a^z$ the aggregate consumption in region z. Then equilibrium in the final good markets is given by (24), in the intermediate goods markets by (26) and (27) and in labor markets by (25).

$$\begin{aligned}
C^z(t) + I^z(t) &= Y^{F,z}(t) \quad \forall z && (24)\\
L^z(t) &= \sum_{a=10}^{75} \mathrm{te}_a^z(t) L_a^z(t) h_a \quad \forall z && (25)\\
X^{D,z} + X^{\star,z} &= Y^{I,z}(t) \quad \forall z && (26)\\
\sum_z M^{\star,z} &= Y^\star && (27)
\end{aligned}$$

It can be easily shown that these equations (24)–(27) together to the previous equilibrium equations are sufficient to describe the real equilibrium of the world economy. As a matter of fact the equilibrium equation of the world producer : $P^\star(t)Y^\star = \sum_z P_I^z(t) \cdot M^{\star,z}(t)$, let define $e^z(t) = \frac{P_I^z(t)}{P^\star(t)}$ is redundant (i.e. *Walras' Law*), but because only relative price are relevant one can also drop (or fix) one absolute price in the model. For calibration purpose we will choose that at each time the price of the final good in the region "North America" will be set to one, so all the value can be seen as expressed in dollars in our model.

Further accounting identities

Notice that implicitly one can recover the standard aggregate budget constraint of national accounts from (24):

$$PIB^z(t) = P_I^z(t) Y^{I,z}(t) = P_f^z(t) Y^{F,z}(t) + P_I^z(t) X^{\star,z}(t) - P^\star(t) M^{\star,z}(t) \tag{28}$$

where : $P_I^z(t) X^{\star,z}(t) - P^\star(t) M^{\star,z}(t)$ is the trade balance of region z expressed in units of the domestic final good.

From CRS and competitive behaviour of the World producer of intermediate good we have $P^\star(t)\left[\sum_z X^{\star,z}(t)^{\frac{\mu-1}{\mu}}\right]^{\frac{\mu}{\mu-1}} = \sum_z P_I^z(t)\cdot M^{\star,z}(t)$, let define $e^z(t) = \frac{P_I^z(t)}{P^\star(t)}$ as the real effective exchange rate of region z (nominal changes are equal to 1) we can rewrite :

$$\sum_z e^z(t)X^{\star,z}(t) - \sum_z X^{W,z}(t) = 0 \tag{29}$$

where $BC^z(t) = e_z(t)X^{\star,z}(t) - M^{\star,z}(t)$ is the trade balance of region z in terms of the world intermediate good and we have :

$$\sum_z BC_{z,t+i} = 0 \tag{30}$$

One can also checks the equilibrium of the world financial assets :

$$\sum_z K^z(t) = \sum_z (L_T^z(t)H^z(t) + \sum_{a=20}^{100} s_a^z L_a^z S_a^z) \tag{31}$$

References

Auerbach, A.J. and L.J. Kotlikoff, *Dynamic Fiscal Policy*, London: Cambridge University Press, 1987.

Bohn, H., *Social Security and Demographic Uncertainty : The Risk Sharing Properties of Alternative Policies*, Working Paper 7030, NBER, Cambridge MA March 1999.

Broer, P. and E. Westerhout, Pension Policies and Lifetime Uncertainty in an Applied General Equilibrium Model, in P. Broer and J. Lassila, editors, *Pension Policies and Public Debt in Dynamic CGE Models*, Heidelberg: Physica-Verlag, 1997, chapter 4, pp. 110–138.

Chauveau, T. and R. Loufir, The Future of Public Pensions in the Seven Major Economies, in P. Broer and J. Lassila, editors, *Pension Policies and Public Debt in Dynamic CGE Models*, Heidelberg: Physica-Verlag, 1997, chapter 2, pp. 16–73.

Docquier, F., P. Liégeois, C. Loupias, and B. Crettez, Vieillissement et inégalités intergénérationnelles en France : une approche par l'équilibre général, *Revue Economique*, Juillet 2002, *53* (4), 767–786.

Gaudemet, J., Les dispositifs d'acquisition à titre facultatif d'annuités viagère en vue de la retraite : une diffusion limitée, *Economie et Statistique*, 2001, *348* (8), 81–106.

Imrohoroglu, S., A quantitative analysis of capital income taxation, *International Economic Review*, May 1998, *39* (2), 307–328.

Ingenue, *Macroeconomic consequences of pension reforms in Europe : an investigation with the Ingenue world model*, Working-Paper 2001-17 (2001-16,2001-07), CEPII (CEPREMAP,OFCE), Paris December 2001.

——, A Long-Term Model for the World Economy, in J Hairault and H. Kempf, editors, *Market Imperfections and Macroeconomic Dynamics*, Boston/London: Kluwer Academic Publishers, 2002a, chapter 3, pp. 51–73.

——, Incidences économiques, politiques et redistributives des réformes de retraites en Europe : une exploration avec le modèle ingenue, *Revue Economique*, Juillet 2002b, *53* (4), 787–808.

Mahieu, R. and B. Sédillot, *Equivalent patrimonial de la rente et souscription de patrimoine compélmentaire*, Working-Paper G2000/09, Insee, Malakoff Juillet 2000.

Obstfeld, Maurice and Kenneth Rogoff, *Foundations of International Macroeconomics*, mit press editor 1996.

Rios-Rull, J., Life-Cycle Economies and Aggregate Fluctuations, *Review of Economic Studies*, 1996, *63*, 465–489.

Yaari, M., Uncertain Lifetime, life Insurance, and the Theory of the Consumer, *review of Economic Studies*, April 1965, *32*, 137–150.